
Heat Equation Problem -Neumann BCs - 11-09-16

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Initialization: Be sure the files *NTGStylesheet2.nb* and *NTGUtilityFunctions.m* is are in the same directory as that from which this notebook was loaded. Then execute the cell immediately below by mousing left on the cell bar to the right of that cell and then typing “shift” + “enter”. Respond “Yes” in response to the query to evaluate initialization cells.

In[10]:=

```
SetDirectory[NotebookDirectory[]];
(* set directory where source files are located *)
SetOptions[EvaluationNotebook[], (* load the StyleSheet *)
  StyleDefinitions -> Get["NTGStylesheet2.nb"]];
Get["NTGUtilityFunctions.m"]; (* Load utilities package *)
```

Previous version - *Heat Equation Problem - Neumann BCs -10-15-16. Much earlier work circa 2008.*

Purpose

I continue to solve heat equation problems from Chapter 3 of *Numerical and Analytical Methods for Scientists and Engineers, Using Mathematica*, Daniel Dubin. This example (p218) considers an insulating slab with von Neumann boundary conditions.

I construct an Association that encapsulates information about this problem. I then apply the function *DSolveHeatEquation* that attempts to solve this problem using the Mathematica function *DSolve*.

In[12]=

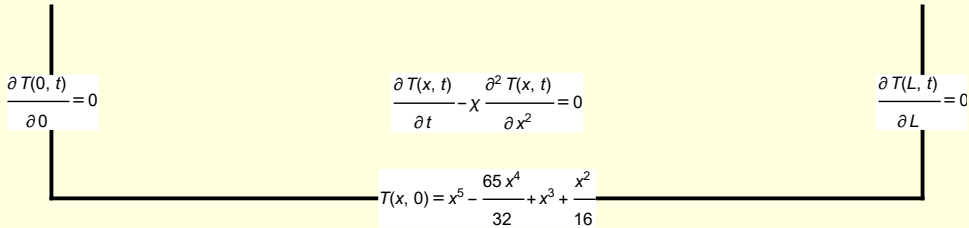
```

A1 =
Module[{description, pde, bcL, bcR, ic, eqns,
  assumptions, substitutions, simplifications, names, values},
  description = "Dubin example, p218 Homogeneous heat equation
    with homogeneous Neumann boundary conditions";
  pde = D[T[x, t], t] - χ D[T[x, t], {x, 2}] == 0;
  bcL = Derivative[1, 0][T][0, t] == 0;
  bcR = Derivative[1, 0][T][L, t] == 0;
  ic = T[x, 0] ==  $\frac{x^2}{16} + x^3 - \frac{65}{32} x^4 + x^5$ ;
  eqns = {pde, bcL, bcR, ic};
  assumptions = {L > 0, χ > 0};
  substitutions = {K[1] → n};
  simplifications = {n ∈ Integers};
  values = {description, pde, bcL, bcR,
    ic, eqns, assumptions, substitutions, simplifications};
  names = {"description", "pde", "bcL", "bcR", "ic", "eqns",
    "assumptions", "substitutions", "simplifications"};
  AssociationThread[names, values]];

Module[{soln, G},
  soln = DSolveHeatEquation[A1];
  AppendTo[A1, "soln" → soln];
  Print@ShowPDESetup[A1];
  A1["soln"]]

```

Dubin example, p218 Homogeneous heat equation with homogeneous Neumann boundary conditions



Out[13]=

$$T[x, t] \rightarrow \frac{1}{96 L} \left(L^3 (2 + 24 L - 39 L^2 + 16 L^3) + 192 \right. \\
 \left. \sum_{n=1}^{\infty} \frac{e^{-\frac{n^2 \pi^2 t \chi}{L^2}} L^3 \left((-1)^n n^4 \pi^4 - 65 (-1)^n L^2 n^2 \pi^2 (-6 + n^2 \pi^2) + 24 L n^2 \pi^2 (2 - 2 (-1)^n + (-1)^n n^2 \pi^2) \right)}{8 n^6 \pi^6} \right)$$

The unusual initial conditions were chosen to satisfy the boundary conditions. In this case involving a homogeneous pde and homogeneous boundary conditions, DSolve can immediately found the solution.

In[14]:=

```
Clear[Tinit, TSoln];
```

```
Tinit[x_] :=  $\frac{x^2}{16} + x^3 - \frac{65}{32} x^4 + x^5$ ;
```

```
TSoln[t_, L_, x_, nMax_] :=
```

$$\frac{1}{96 L} \left(L^3 (2 + 24 L - 39 L^2 + 16 L^3) + 192 \right.$$

$$\left. \sum_{n=1}^{nMax} e^{-\frac{n^2 \pi^2 t x}{L^2}} L^3 \frac{((-1)^n n^4 \pi^4 - 65 (-1)^n L^2 n^2 \pi^2 (-6 + n^2 \pi^2) + 24 L n^2 \pi^2 (2 - 2 (-1)^n + (-1)^n n^2)}{8 n^6 \pi^6} \right.$$

```
    // Activate
```

With insulating boundary conditions, the initial temperature profile flattens.

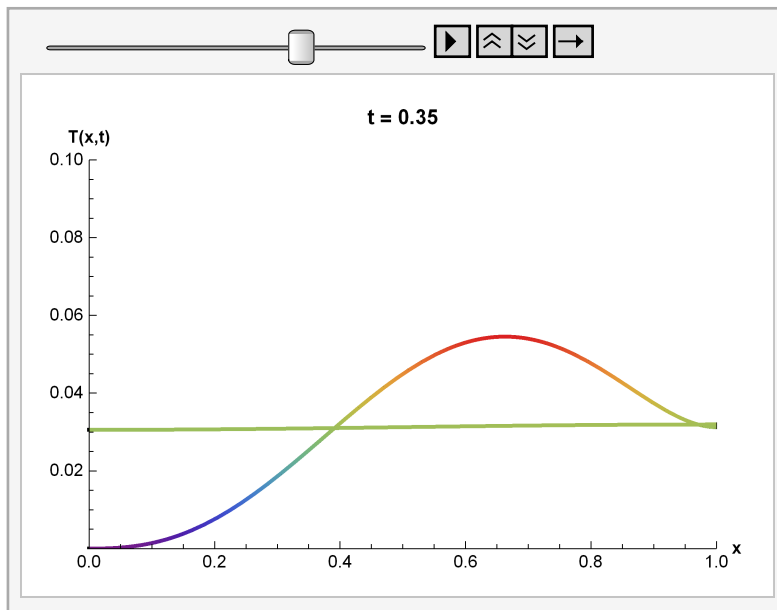
In[17]:=

```
Module[{L = 1, x = 1, nMax = 10, frames},
```

```
frames = MakeFrame[#, L, x, nMax] & /@ Range[0, 0.5, 0.05];
```

```
ListAnimate[frames]]
```

Out[17]=



In[3]:=

```

Clear[MakeFrame];
MakeFrame[t_, L_, x_, nMax_] :=
Module[{lab},
  lab = Stl@StringForm["t = ``", NF2@t];
  Plot[{TSoln[t, L, x, nMax], Tinit[x]}, {x, 0, 1}, PlotLabel → lab,
  AxesLabel → {Stl["x"], Stl["T(x,t)"]}, PlotStyle → Thick, ColorFunction →
  Function[{x, y}, ColorData["Rainbow"][y]], PlotRange → {{0, 1}, {0, 0.1}}]

```

Functions

In[5]:=

```

Clear[ShowPDESetup];
ShowPDESetup[A_] := Module[{top = 1.0, right = 1.0,
  boundaries, labels, textInterior, textIC, textBCL, textBCR},
  boundaries = Line /@ {{{0, 0}, {right, 0}},
  {{0, 0}, {0, top}}, {{right, 0}, {right, top}}};
  labels = Text[PhysicsForm[A[#[[1]]], #[[2]]] & /@
  {"pde", {right/2, top/2}}, {"ic", {right/2, 0.0}},
  {"bcL", {0.0, top/2}}, {"bcR", {right, top/2}}];
  Graphics[{Directive[Black, Thick], boundaries, labels}, Axes → False,
  AspectRatio → 0.25, ImageSize → 500, PlotLabel → Stl[A["description"]]]

```

In[6]:=

```

Clear[DSolveHeatEquation];
DSolveHeatEquation[A_] :=
Module[{soln},
  soln = DSolve[A["eqns"], T[x, t], {x, t}, Assumptions → A["assumptions"]][[1, 1]];
  soln = soln //. A["substitutions"];
  soln = Simplify[soln, A["simplifications"]];
  soln]

```