
TB 13-16 Mean Free Path 03-12-18

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Initialization: Be sure the file *NTGUtilityFunctions.m* is in the same directory as that from which this notebook was loaded. Then execute the cell immediately below by mousing left on the cell bar to the right of that cell and then typing “shift” + “enter”. Respond “Yes” in response to the query to evaluate initialization cells.

```
SetDirectory[NotebookDirectory[]];  
(* set directory where source files are located *)  
Get["NTGUtilityFunctions.m"]; (* Load utilities package *)
```

Purpose

This is the 9th in a series of notebooks in which I work through material and exercises in the magisterial new book *Modern Classical Physics* by Kip S. Thorne and Roger D. Blandford. If you are a physicist of any ilk, BUY THIS BOOK. You will learn from a close reading and from solving the exercises.

Exercise 13.16 Problem: Mean Free Path

Estimate the collisional mean free path of the air molecules around you. Hence verify the estimate for the kinematic viscosity of air given in Table 13.2

Solution

This is a straightforward calculation. The mean free path for a gas is

$$\lambda = \frac{1}{n\sigma}$$

(1)

where n is the number density and σ is the collisional cross-section

From https://chem.libretexts.org/Core/Physical_and_Theoretical_Chemistry/Kinetics/Modeling_Reaction_Kinetics/Collision_Theory/Collisional_Cross_Section

Although the collisional cross section of a particle can be calculated, it is usually not used on its own (Table 1.1). Instead, it is a component of more complex equations for calculating collision frequency and collision theory.

Figure 1.1: Table of Cross-Sections for Common Gases

Molecule	Cross-Section (nm ²)
Ar	0.36
C ₂ H ₄	0.64
C ₆ H ₆	0.88
CH ₄	0.46
Cl ₂	0.93
CO ₂	0.52
H ₂	0.27
He	0.21
N ₂	0.43
Ne	0.24

we see that $\sigma_{N_2} = 0.43 \text{ nm}^2$. Since N₂ is 80% of the atmosphere, I will take σ_{N_2} as a representative cross-section for air molecules.

Calculation of the mean free path

```
Module[{ρATM = 1.2 kg/m³, mN2 = 2 × 14 amu, σN2 = 0.43 nm²,
  nATM, numberDensityEqn, meanFreePathEqn, meanFreePath, soln},
  numberDensityEqn = nATM == ρATM/mN2 /. amu → 1.66 × 10-27 kg;
  meanFreePathEqn = λMFP ==  $\frac{1}{nATM \sigma N2}$  /. Sol[numberDensityEqn, nATM] /. m → 109 nm;
  meanFreePath = Sol[meanFreePathEqn, λMFP] ]
```

$\lambda_{MFP} \rightarrow 90.0775 \text{ nm}$

Calculation of the kinematic viscosity

$$v = \frac{1}{3} v_{th} \lambda$$

(2)

```

Module[{λ = 90.07 nm, mN2 = 2 × 14 amu, kB = 1.38 × 10-23 J/degreesK,
  Troom = 295 degreesK, thermalVelocity, thermalVelocityEqn, kinematicViscosityEqn},
  thermalVelocityEqn =  $\frac{3}{2}$  kB Troom == mN2 vth2;
  thermalVelocity =
  Solve[thermalVelocityEqn, vth] [[2, 1]] /. amu → 1.66 × 10-27 kg /. J → kg m2/sec2 //
  PowerExpand;
  Print[thermalVelocity];
  kinematicViscosityEqn =  $\nu$  ==  $\frac{1}{3}$  vth λ /. thermalVelocity /. nm → 10-9 m]

```

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

$$v_{th} \rightarrow \frac{362.463 \text{ m}}{\text{sec}}$$

$$\nu == \frac{0.0000108823 \text{ m}^2}{\text{sec}}$$

Table 13.2 gives $\nu = 10^{-5} \text{ m}^2/\text{sec}$