Initialization: Be sure the file `NTGUtilityFunctions.m` is in the same directory as that from which this notebook was loaded. Then execute the cell immediately below by mousing left on the cell bar to the right of that cell and then typing “shift” + “enter”. Respond “Yes” in response to the query to evaluate initialization cells.

```mathematica
SetDirectory[NotebookDirectory[]]; (* set directory where source files are located *)
Get["NTGUtilityFunctions.m"]; (* Load utilities package *)
```

Purpose

This is the 12th in a series of notebooks in which I work through material and exercises in the magisterial new book *Modern Classical Physics* by Kip S. Thorne and Roger D. Blandford. If you are a physicist of any ilk, BUY THIS BOOK. You will learn from a close reading and from solving the exercises.

Exercise 13.19  Example: Pulsatile Blood Flow

Consider the pulsatile flow of blood through one of the body’s larger arteries. The pressure gradient \( \frac{dP}{dz} = P'(t) \) consists of a steady term plus a term that is periodic with the period of the heart’s beat.

(a) Assuming laminar flow with \( \textbf{v} \) pointing in the \( z \) direction and being a function of radius and time, \( \textbf{v} = v(\rho, t) \textbf{e}_z \), show that the Navier-Stokes equation reduces to \( \frac{\partial \textbf{v}}{\partial t} = -\nabla P' + v \nabla^2 \textbf{v} \).

(b) Explain why \( v(\rho, t) \) is the sum of a steady term produced by the steady (time-independent) part of \( P' \), plus terms at angular frequencies \( \omega_0, 2\omega_0, \ldots \), produced by parts of \( P' \) that have these frequencies. Here \( \omega_0 = \frac{2\pi}{\text{heart's beat period}} \).
I will solve this problem in the context of a model for blood flow through an artery.

Consider Navier-Stokes equation with gravity assumed negligible

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \cdot \mathbf{v} = -\frac{\nabla P}{\rho} + \frac{1}{\rho} \frac{\partial \sigma_{ik}}{\partial x_k} \tag{1}
\]

For incompressible flow, this can be written

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \cdot \mathbf{v} = -\frac{\nabla P}{\rho} + \alpha \nabla^2 \mathbf{v} \tag{2}
\]

where I use \(\alpha = \eta/\rho\) to represent the kinetic viscosity. The Mathematica font for “nu” looks too much like “vee”.

For laminar flow in the z-direction, \(\mathbf{v} = v(r) \mathbf{\hat{z}}\)

\[
\frac{\partial v_z(r, t)}{\partial t} = -\frac{1}{\rho} \frac{\partial P(z, t)}{\partial z} + \nu \nabla^2 v_z(r, t) \tag{3}
\]

because \((\mathbf{v} \cdot \nabla) \cdot \mathbf{v} = v_z(r) \partial v_z(r)/\partial z = 0\)

This is the equation called for in part (a) of TB exercise (13.19)
In general, the time-dependence of pressure gradient term that drives the flow is modeled by a Fourier series with $\omega_0$ being the frequency of a heart beat.

$$\frac{1}{\rho} \frac{\partial P(r, t)}{\partial z} \equiv \mathcal{P} \sum_{k=0}^{\infty} \epsilon_n e^{i k \omega_0 t}$$

The $k = 0$ term corresponds to a time-independent pressure driver. This is the explanation sought in part (b) of exercise TB13-19.

Note

Laplacian[f[R], {R, θ, z}, "Cylindrical"]

So the pde to be solved is

$$\frac{\partial v_z(r, t)}{\partial t} = -\mathcal{P} \sum_{k=0}^{\infty} \epsilon_n e^{i k \omega_0 t} + \alpha \left( \frac{1}{r} \frac{\partial v_z(r, t)}{\partial r} + \frac{\partial^2 v_z(r, t)}{\partial r^2} \right)$$

Like all good physicists, I seek a dimensionless form

$$w[2] = w[1] \gamma v_z \rightarrow \text{Function}[(r, t), \gamma v_0 V[r/a, t/t_0]]$$

The natural scale length is the radius of of an artery, $a$

$$w[3] = w[2] \gamma r \rightarrow aR \gamma t \rightarrow Tt_0$$

$$w[4] = \text{MapEqn}\left[\left(\gamma t_0 / v_0\right) \& w[3]\right] \gamma \text{ExpandAll}$$
Choose the time scale to be the diffusion time across the arterial radius

\[
def[t_0] = \frac{t_0 \nu}{a^2} = 1
\]

\[
t_0 \nu \quad \frac{1}{a^2} = 1
\]

\[
\]

\[
V^{(1,1)}[R, \theta] = - \sum_{k=0}^{n} e^{i k T t_0} e^{i k \theta} \epsilon_k e_k + V^{(1,0)}[R, \theta] + V^{(2,0)}[R, \theta]
\]

Define a dimensionless flow speed

\[
def[v_0] = \frac{t_0 \nu}{v_0} = 1
\]

\[
t_0 \nu \quad \frac{1}{v_0} = 1
\]

\[
\]

\[
V^{(1,1)}[R, \theta] = - \sum_{k=0}^{n} e^{i k T \omega_0} e_k + V^{(1,0)}[R, \theta] + V^{(2,0)}[R, \theta]
\]

The characteristic dimensionless oscillation frequency is defined

\[
def[\Omega] = \Omega = \omega_0 t_0
\]

\[
\Omega = t_0 \omega_0
\]

where \( \Omega = 1/(2 \pi) \) is a canonical value corresponding to a pulse rate of 60/minute.

\[
w[7] = w[6] / \text{Sol}[\text{def[\Omega]}, \omega_0]
\]

\[
V^{(1,1)}[R, \theta] = - \sum_{k=0}^{n} e^{i k \Omega T} \epsilon_k e_k + V^{(1,0)}[R, \theta] + V^{(2,0)}[R, \theta]
\]

This constitutes the starting equation for the analysis

Seek a solution having the form

\[
V(R, \theta) = V_0(R, \theta) + \sum_{k=0}^{n} e_k V_k(R) e^{i k \Omega T}
\]

Note that the \( k = 0 \) term corresponds to a constant driver
\[ w[8] = w[7] \cdot \sum_{k=0}^{n} e^{i k T} \epsilon_k \to 1 \cdot V \to \text{Function}[(R, T), V_0(R)] \]
\[ \theta = -1 \cdot \frac{V_0'(R)}{R} + V_0''[R] \]

The solution of this equation corresponds to Poiseuille flow.

\[ w[9] = \text{DSolve}[[w[8], V_0[1] = \theta, V_0'[\theta] = \theta], V_0[R], R][1, 1] \]
\[ V_0[R] \to \frac{1}{4} (-1 + R^2) \]

For future use, I define the function

\[ \text{Clear}[\text{VPoiseuille}]; \]
\[ \text{VPoiseuille}[\_][R_] := \frac{1}{4} (-1 + R^2) \]

The general oscillatory problem is solved by separation of variables, and the particular ansatz

\[ w[10] = w[7] \cdot \sum_{k=0}^{n} e^{i k T} \epsilon_k \to e^{i k T} \epsilon_k \cdot \cdot \cdot V \to \text{Function}[(R, T), e_k V_k[R] \text{Exp}[I k \Omega T]] \]
\[ i \cdot e^{i k T} k \cdot e \cdot \epsilon_k V_k[R] = -e^{i k T} \epsilon_k + \frac{e^{i k T} \epsilon_k V_k'[R]}{R} + e^{i k T} \epsilon_k V_k''[R] \]

\[ w[11] = \text{MapEqn}[\{(\text{Simplify}[#/(e_k \text{Exp}[I k T \Omega])] \} \&. w[10]] \]
\[ i \cdot k \cdot \Omega \cdot V_k[R] = -1 + \frac{V_k'[R]}{R} + V_k''[R] \]

The solution is

\[ w[12] = \text{DSolve}[w[11], V_k[R], R][1, 1] \]
\[ V_k[R] \to \frac{1}{k \Omega} + \text{BesselJ}[\theta, (-1)^{3/4} \sqrt{k} R \sqrt{\Omega}] C[1] + \text{BesselY}[\theta, (-1)^{3/4} \sqrt{k} R \sqrt{\Omega}] C[2] \]

The condition that the solution be well behaved at \( R = 0 \) requires \( C[2] = 0 \)

\[ w[13] = w[12] \cdot C[2] \to 0 \]
\[ V_k[R] \to \frac{1}{k \Omega} + \text{BesselJ}[\theta, (-1)^{3/4} \sqrt{k} R \sqrt{\Omega}] C[1] \]

The boundary condition is \( V_k(R = 1) = 0 \)
\[ w[14] = (w[13] \text{ // RE} \rightarrow . R \rightarrow 1 / . V_k[1] \rightarrow 0 \]

\[ \theta = \frac{\hat{v}}{k \Omega} + \text{BesselJ}[\theta, (-1)^{3/4} \sqrt{k \cdot \sqrt{\Omega}}] \cdot C[1] \]

Thus

\[ \text{Sol}[w[14], C[1]] \]

\[ C[1] \rightarrow -\frac{\hat{v}}{k \Omega \cdot \text{BesselJ}[\theta, (-1)^{3/4} \sqrt{k \cdot \sqrt{\Omega}}]} \]


\[ V_k[R] \rightarrow \frac{\hat{v}}{k \Omega} \left( 1 - \frac{\text{BesselJ}[\theta, (-1)^{3/4} \sqrt{k \cdot \sqrt{\Omega}}]}{\text{BesselJ}[\theta, (-1)^{3/4} \sqrt{k \cdot \sqrt{\Omega}}]} \right) \]

With less elucidation, the original ode could have been solved immediately

\[ w[16] = \text{DSolve}[\{w[11], V_k[1] = 0, V_k'[\theta] = 0\}, V_k[R], R][1, 1] \]

\[ V_k[R] \rightarrow \frac{\hat{v}}{k \Omega} \left( 1 - \frac{\text{BesselJ}[\theta, (-1)^{3/4} \sqrt{k \cdot \sqrt{\Omega}}] - \text{BesselJ}[\theta, (-1)^{3/4} \sqrt{k \cdot \sqrt{\Omega}}]}{\text{BesselJ}[\theta, (-1)^{3/4} \sqrt{k \cdot \sqrt{\Omega}}]} \right) \]

To reduce the size of the expressions in the analysis to follow, I introduce the parameter

\[ \text{def}[\beta] = \beta = (-1)^{3/4} \sqrt{k \cdot \sqrt{\Omega}} \]

\[ \beta = (-1)^{3/4} \sqrt{k \cdot \sqrt{\Omega}} \]

\[ w[17] = w[16] / . \text{Sol}[\text{def}[\beta], \Omega] / . \text{PowerExpand} / . \text{Expand} / . \text{RE} \]

\[ V_k[R] \rightarrow \frac{1}{\beta^2} - \frac{\text{BesselJ}[\theta, R \beta]}{\beta^2 \text{BesselJ}[\theta, \beta]} \]

\[ w[18] = w[17][1] = \text{Collect}[w[17][2], \beta] \]

\[ V_k[R] \rightarrow \frac{1 - \frac{\text{BesselJ}[\theta, R \beta]}{\text{BesselJ}[\theta, \beta]}}{\beta^2} \]

The solution will have a different character for large and small \( \beta \).
### Module

```math
Module[(Vk),
Vk[R_, β_] := Re[1 - BesselJ[0, R, β]/BesselJ[0, β]]/β;
Plot[{{Vk[R, 0.1], Vk[R, 1], Vk[R, 10], Vk[R, 100], VPoiseuille[R]}, {R, 0, 1},
   PlotStyle -> {Black, Blue, Green, Orange, Directive[RED, Dashed]},
   PlotLegends -> {Stl["β = 1/10"], Stl["β = 1"], Stl["β = 10"],
    Stl["β = 100"], Stl["Poiseuille"]}, AxesLabel -> {Stl["R"], Stl["Vk"]}]]
```

Note that for \( \beta \lesssim 1 \), the shape of the time-dependent flow is similar to that of Poiseuille. For \( \beta \gg 1 \) the time-dependent flow has a quite different shape.

The analytical limiting form for \( \beta \ll 1 \) is easily obtained with

```math
w[19] = Normal@Series[1 - BesselJ[0, R, β]/BesselJ[0, β], {β, 0, 1}]
\[
\frac{1}{4} (-1 + R^2)
\]
```

The \( \beta \gg 1 \) limit requires a bit more care

```math
w[20] = Normal@Series[(BesselJ[0, x]), {x, ∞, 0}]
\[
\sqrt{\frac{2}{\pi}} \sqrt{\frac{1}{x} \cos \left( \frac{x}{4} \right)}
\]
The leading order approximation is quite good for \( \beta = 10 \).

Exercise TB 13-19 (c) calls for determining the character of the flow as a function of frequency.
The quantity $W$ is known as the Womersley number

$$W = \sqrt{\frac{a^2 \omega_0}{\nu}}$$

From http://wwwf.imperial.ac.uk/~ajm8/BioFluids/lec1114.pdf, I find a representative value of $W$ appropriate for blood flow.

In a physiological context, $\alpha$ is known as the Womersley number, but similar parameters are well known in other contexts by other names.

In the aorta, with $a = 0.015$, $2\pi/\omega = 1$ and $\nu = \mu/\rho = 4 \times 10^{-6}$ we have

$$\sqrt{\frac{R^2 \omega_0}{\nu}}$$

18.7997

So a representative value is $W = 19$ and the $\beta >> 1$ expansion is the relevant one for the present context of arterial flow. In more general contexts, this quantity is called the Strouhal number.

I use this information to consider a simplified form of $V_k$

$$V_k[R] = \frac{1}{k t_0 \omega_0} \left( \frac{\text{BesselI}[\theta, (-1)^{1} \sqrt{k t_0 \omega_0}]}{\sqrt{2}} \right)$$

As suggested in part (c), consider the frequency $\omega = k \omega_0$
Introduce the dimensionless parameter \( f = \omega t_0 \)

\[
\begin{align*}
\omega_{24} &= \omega_{23} / \omega_0 \rightarrow \omega / k / \text{PowerExpand} \\

V_k[R] &= \frac{i \left( 1 - \frac{\text{BesselJ}[0, \frac{1-i t_0 \sqrt{\omega}}{\sqrt{2}}]}{\text{BesselJ}[0, \frac{1+i t_0 \sqrt{\omega}}{\sqrt{2}}]} \right)}{t_0 \omega} \\
\end{align*}
\]

So — the answer to Exercise TB13-19 (3) is that the central flow is very similar to Poiseuille flow (order -0.25) for low frequency \( f < \ll 1 \) and quite different for \( f >> 1 \).

With regard to Exercise TB13-19 part (d), the desired result just requires some manipulation of previous results.
\[ V(R, T) = \sum_{k=0}^{n} \text{Re}[V_n(R) e^{i k \Omega T}] = V_0 + \sum_{k=1}^{n} \epsilon_k \text{Re}[V_n(R) e^{i k \Omega T}] \]

Recall

\[ w[9] // \text{RE} \]

\[ V_0[R] = \frac{1}{4} (-1 + R^2) \]

Recall the previous results

\[ w[18] \]

\[ V_k[R] = \frac{1 - \text{BesselJ}[0, R \beta]}{\text{BesselJ}[0, \beta]} \]

\[ w[22] \]

\[ \beta = (-1)^{3/4} \sqrt{k} \left( \frac{a^2 \omega_0}{\nu} \right) \]

\[ \text{def[W]} \]

\[ W = \sqrt{\frac{a^2 \omega_0}{\nu}} \]

Then

\[ w[26] = w[18] / \text{Sol[def[\beta], \beta]} / \text{Sol[def[\Omega], \Omega]} / \text{Sol[def[t0], t0]} / \text{Sol[def[W], a]} // \text{PowerExpand} \]

\[ V_k[R] = \frac{1}{k W^2} \left( \frac{1 - \text{BesselJ}[0, \frac{1}{\sqrt{2}} \sqrt{k W}]}{\text{BesselJ}[0, \frac{1}{\sqrt{2}} \sqrt{k W}]} \right) \]

The combined solution is
\[ w[27] = \]

\[ \frac{1}{4} \left( -1 + R^2 \right) + \text{Inactive} \left[ \text{Sum} \left[ \text{Re} \left[ \frac{1 - \text{BesselJ} \left[ \theta, \left( \frac{1+i}{} \frac{k W}{1} \right] \right]}{k W^2} \text{Exp}\left[ i k \omega_0 t \right]} \right], \{k, 1, n\} \right] \]

\[ e^{i k t + \omega_0} \left( \frac{1 - \text{BesselJ} \left[ \theta, \left( \frac{1+i}{} \frac{k W}{1} \right] \right]}{k W^2} \text{Exp}\left[ i k \omega_0 t \right]} \right] \]

which is equivalent to the result TB 13.85

For part (e) I construct

\[
\text{Clear}\left[ \text{Vk}, \text{Pk} \right];
\text{Vk}[R, t, k, \omega_0, W, \epsilon_k] :=
\frac{i}{k W^2} \text{Re}\left[ \frac{1 - \text{BesselJ} \left[ \theta, \left( \frac{1+i}{} \frac{k W}{1} \right] \right]}{k W^2} \text{Exp}\left[ i k \omega_0 t \right]} \right];
\text{Pk}[t, k, \omega_0] := \text{Re}\left[ \text{Exp}\left[ i k \omega_0 t \right] \right]
\]

When the Womersley number \( W << 1 \) (diffusion time is much longer than oscillation time), there is balance between the driving term and the viscous force. The velocity is in phase with the driving term.

\[
\text{Module}\left[ \{ k = 1, \omega_0 = 1/(2 \pi), R = 0.1, W = 0.1, \epsilon_k = 10 \},
\text{Plot}\left[ \{ \text{Pk}[t, k, \omega_0], \text{Vk}[R, t, k, \omega_0, W, \epsilon_k] \},
\{t, \theta, 50\}, \text{PlotStyle} \rightarrow \{ \text{Black}, \text{Blue} \}, \text{PlotLegends} \rightarrow
\{ \text{Stil"pressure driver"}, \text{Stil"fluid response"} \}, \text{AxesLabel} \rightarrow \{ \text{Stil"t"}, \text{""} \} \right]
\]

When the Womersley number \( W >> 1 \) (diffusion time is much slower than oscillation time), there is
balance between the driving term and the inertial term. The velocity is out of phase with the driving term.

\[
\text{Module}\left[\{k = 1, \omega \theta = 1/(2\pi), R = 0.1, W = 5, \epsilon k = 10\}, \right.
\]
\[
\text{Plot}\left[\{P_k[t, k, \omega \theta], V_k[R, t, k, \omega \theta, W, \epsilon k]\}, \{t, 0, 50\}, \text{PlotStyle} \to \{\text{Black, Blue}\}, \right.
\]
\[
\text{PlotLegends} \to \{\text{Stl["pressure driver"]}, \text{Stl["fluid response"]}\}, \]
\[
\text{AxesLabel} \to \{\text{Stl["t"], ""}\} \}
\]

I consider the boundary layer that forms near the edge of an artery when \(W \gg 1\)

\[
\text{Module}\left[\{k = 1, \omega \theta = 1/(2\pi), R = 0.1, W = 10, \epsilon k = 10\}, \right.
\]
\[
\text{Plot}\left[\{V_k[R, 0, k, \omega \theta, W, \epsilon k], 1/4 \left(-1 + R^2\right)\}, \{R, 0, 1\}, \right.
\]
\[
\text{AxesLabel} \to \{\text{Stl["R"]}, ""\}, \text{PlotStyle} \to \{\text{Black, Blue, Green, Orange, Directive[Black, Dashed]}\}, \text{PlotLegends} \to \{\text{Stl["T = 0"]}, \text{Stl["T = 2.5"]}, \text{Stl["T = 5"]}, \text{Stl["T = 10"]}, \text{Stl["Poiseuille"]}\} \]

Consider the \(T = 0\) flow.
\[ w[26] = V_k[R, \theta, k, \omega_0, W, ek] / k \to 1 \]

\[ e_k \left( 1 - \frac{\text{BesselJ}[\theta, \sqrt{1 - \frac{(1 - i) RW}{\sqrt{2}}}]}{\text{BesselJ}[\theta, \sqrt{1}]} \right) - \text{Im}\left[ \frac{1 - \frac{\text{BesselJ}[\theta, \sqrt{1 - \frac{(1 - i) W}{\sqrt{2}}}]}{\text{BesselJ}[\theta, \sqrt{1}]} W^2}{W^2} \right] \]

and focus on radial dependence

\[ w[27] = \left( 1 - \frac{\text{BesselJ}[\theta, \sqrt{1 - \frac{(1 - i) RW}{\sqrt{2}}}]}{\text{BesselJ}[\theta, \sqrt{1}]} \right) \frac{1 - \frac{\text{BesselJ}[\theta, \sqrt{1 - \frac{(1 - i) W}{\sqrt{2}}}]}{\text{BesselJ}[\theta, \sqrt{1}]} W^2}{W^2} \]

\[
\text{Module}[\{F\},
F[R_, W_] := -\text{Im}\left[ 1 - \frac{\text{BesselJ}[\theta, \sqrt{1 - \frac{(1 - i) RW}{\sqrt{2}}}]}{\text{BesselJ}[\theta, \sqrt{1}]} \right];
\]

\[
\text{Plot}[\{F[R, 1], F[R, 10], F[R, 20], F[R, 50]\}, \{R, \theta, 1\},
\text{AxesLabel} \to \{\text{Stl}["R"], ""\}, \text{PlotStyle} \to \{\text{Black}, \text{Blue}, \text{Green}, \text{Orange}\},
\text{PlotLegends} \to \{\text{Stl}["W = 5"], \text{Stl}["W = 10"], \text{Stl}["W = 20"], \text{Stl}["W = 50"]\},
\text{PlotLabel} \to \text{Stl}["Boundary layer becomes pronounced at high W"]]
\]

I have not previously considered analytical solutions of boundary layers in fluids so I will content myself with a geometrical/numerical answer to TB13-19 part (e). Later, I intend to return and obtain an analytical result.

I define a function to estimate the width of the boundary layer
Clear[FRadial, EstimateBoundaryLayerThickness];

FRadial[R_, W_] := -Im \left[ \frac{1 - \frac{1}{\sqrt{2}} \text{Besselj}_0(\text{ik} R W)}{\sqrt{2}} \right];

(* I estimate the thickness of the boundary layer to be the half-width of the negative pulse near the edge of the artery. *)
EstimateBoundaryLayerThickness[W_, RMinGuess_] :=
Module[{result, min, RVal, RHalfInner, RHalfOuter, δBoundaryLayer, F1, F2},
(* find minimum near $R = 1$ *)
result = FindMinimum[{FRadial[R, W], 0.7 < R < 0.999}, {R, RMinGuess}];
{min, RVal} = {result[[1]], result[[2, 1, 2]]};
(* find the Rinner value at which $F = \text{min/2}$ *)
RHalfInner = FindRoot[FRadial[R, W] == min/2, {R, RVal - 0.02}][[1, 2]];
(* find the Router value at which $F = \text{min/2}$ *)
RHalfOuter = FindRoot[FRadial[R, W] == min/2, {R, RVal + 0.02}][[1, 2]];
(* estimate $δ$ as Router - Rinner *)
δBoundaryLayer = RHalfOuter - RHalfInner
]

I calculate $δ(W)$ numerically and then obtain a functional fit

Module[{values, fit, valuesFit, aFit, bFit, lab},
values = Table[{W, EstimateBoundaryLayerThickness[W, 0.95]}, {W, 10, 50, 5}];
fit = FindFit[values, a W^b, {a, b}, W];
valuesFit = Interpolation@Table[{W, a W^b} /. fit, {W, 10, 50, 5}];
lab = Stl[StringForm["fitting function = a x^b a = ``, b = ``", a, b] /. fit;
Plot[valuesFit[W], {W, 10, 50}, AxesLabel -> {Stl["W"], Stl["δ"]}],
Epilog -> {Red, PointSize[0.02], Point /@ values}, PlotLabel -> lab]
]

The numerical result strongly suggest $δ \sim 1/W$. 

References
On pulsatile flow
https://www.youtube.com/watch?v=HFUSnqaFio0
https://www.youtube.com/watch?v=b029wRQnZwI&list=PLbMVogVj5nJRsXU2WZyMTIDq6tPCjIQB3&index=17
	nice summary of solving inhomogeneous PDE using eigenfunction expansions
https://www.youtube.com/watch?v=HoLweaiYG7g

Detailed analysis of Womersley flow
https://www.google.com/search?q=womersley+flow&oq=womersley&aqs=chrome.1.69i57j0i5.5363j0j8&sourceid=chrome&ie=UTF-8

Wikipedia has a nice treatment of pulsatile flow
https://en.wikipedia.org/wiki/Pulsatile_flow

Mathematica demonstration
http://demonstrations.wolfram.com/PulsatileFlowInACircularTube/

Nice physical pictures in the context of blood flow
http://www.mate.tue.nl/people/vosse/docs/cardio.pdf